AP Calculus Explorations for Teachers

By Lin McMullin

DIFFERENTIAL EQUATIONS

Differential equations are tested every year. The actual solving of the differential equation is usually the main part of the problem, but it is accompanied by a question about its slope field or a tangent line approximation of some sort. BC students may also be asked to approximate using Euler's Method. Large parts of the BC questions are often suitable for AB students and contribute to the AB subscore of the BC exam.

Often the analytic solution of the differential equation is the one single part of any problem that is worth the most points (5 – 7) on the exam.

What students should be able to do:

- Find the *general solution* of a differential equation using the method of separation of variables (this is the *only* method tested).
- Find a *particular solution* using the initial condition to evaluate the constant of integration – initial value problems (IVP). IVPs also appear in particle motion problems and rate/accumulation problems with equations of the form $\frac{dy}{dx} = f(x)$; with solution $y(x) = y(a) + \int_a^x f(t)\,dt$.
- Understand that a solution of a differential equation is a function (not a number) and if it and its derivative are substituted into the given differential equation the resulting equation is true. This may be part of doing the problem even if solving the differential equation is not required (see 2002 BC 5(c)).
- Check a proposed solution by substituting into the differential equation.
- Consider the domain of the solution (see 2006 AB 6 (b)). Also see "Solving Separable Differential Equations: Antidifferentiation and Domain Are Both Needed" by David Loman at AP Central > AB Calculus homepage or http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/37292.html
- Growth-decay problems: The change in the amount is proportional to the amount: $\frac{dy}{dt} = ky$
- Slope fields
  - Sketch a slope field
  - Sketch a *particular solution* on a slope field
  - Match a slope field with a differential equation
  - Match a slope field with a solution
  - Multiple choice 1998 BC #24 and 2003 BC #14
  - Interpret a slope field
- For BC only: Use Euler's Method to approximate a solution.
- For BC only: use the method of partial fractions to find the antiderivative after separating the variables.
- For BC only: understand the logistic growth model, its asymptotes, meaning, etc.

Technology

So far all differential equation problems have been on the no calculator part of the exams. Technology can and should be used in class and by your students to help them understand differential equations.

In real life slope fields (aka direction fields) are drawn by computers and not by hand.

- Winplot is very good at producing slope fields and illustrating particular solutions and the effect of the constant. The slope fields are much better than those produced by graphing calculator screen captures.
- Using Winplot, draw the general solution over the slope field and animate the constant to see how the constant affects the solutions.
- Graphing calculators
    - TI83/84 models. Do not include a built in slope field program but these can be added. There are also programs for doing the Euler method computations and graphing the Euler method solution approximation.
    - TI89/TI92 Plus/Voyage 200 all have built in slope field programs and will draw Euler method approximations.

1. Exercise 1: 2004 AB 6 (No calculator allowed)

The Stem

- Given a separable differential equation and a graph with points marked. Is the anything unusual or deep here?

The parts

- Part (a) is a typical sketch the slope field question. Sketching a slope field is a good way to introduce the topic. What do students need to know to do this?

- How should students handle zero slope, undefined slopes, and slopes that reduce to indeterminate forms?

- Part (b) asks where the slopes are positive. How can a student tell?

- Part (c) requires the analytic solution. What method should be attempted?

- When should the "+ *C*" be used? Is it possible to use it too late?

- What do you tell students who want to include a constant of integration on both sides? What common mistake can the make by putting a "+ *C*" on both sides?

- Absolute value symbols: When and why are they needed? When do they "go away"?

Discuss the scoring standard for this question.

Exercise 2: 2002 BC 5 Parts (a), (c), and (d) are suitable for AB classes. (No calculator allowed.)

The stem

- What is unusual about this differential equation?

The parts

- In part (a) a slope field is given (printed in the answer book). Students are asked to draw 2 particular solutions on the slope field. What 3 things should a student do when graphing a particular solution on the slope field?

- Part (b) Euler's Method. What needs to be shown?

- Part (c) There are (at least) two ways to approach this part. Describe them.

- Both methods require students to understand an import idea about an alleged solution of a differential equation. What is it? Why is it important?

- Part (d) asks about a third general solution. Some students tried to argue that this solution has a maximum value at the origin based on this slope field. Is it possible to justify the maximum based on the slope field? Why or why not?

- Other students tried to argue using the first derivative test. Is this possible? Why or why not?

- How can you justify the existence of a maximum at (0,0)? What 2 concepts (somewhat from "out of the blue") are necessary to do this?

Discuss the scoring standard for this question.

3. Exercise 3: Now you're on your own. Do 2006 AB 5. Timed: 15 minutes.
Discuss the scoring standard for this question.

4. Brainstorm:

What can you have your STUDENTS doing on a regular basis so that they will master these concepts?

1.

2.

3.

4.

Index to AP Free-response questions Topic 5.

The questions themselves can be found at AP Central. Click on "The Exams" and then on "Exam Questions" then "Calculus AB" or "Calculus BC" in the table. or go to http://apcentral.collegeboard.com/apc/members/exam/exam_questions/1997.html

Topic 7: Differential Equations		
Year & Question	**GCA**	**Comments**
1998 AB4	Yes	IVP, equation of tangent line
1998 BC4	Yes	IVP field, Euler, solve
1999 BC6	Yes	Tangent line with approximation, Euler, FTC with "impossible integrand" in (c)
2000 AB6	No	IVP, find domain and range of solution.
2000 BC6	No	Slope field with analysis, IVP, range
2001 AB6	No	Second derivative by implicit differentiation of y', IVP
2001 BC5	No	Improper integral, Euler, IVP
2002 BC5	No	Draw solution on given slope filed, Euler, max/min, check solution, 2-DT,
2002 AB5-BC5 B	No	2-DT, IVP
2003 AB5-BC5	No	Related Rate, IVP
2003 BC6 (c)	No	From power series check solution of differential equation
2003 AB 6 B	No	Second derivative by implicit differentiation of y', IVP
2004 AB6	No	Draw slope field, generalize, IVP
2004 BC5	No	Logistics Diff Eq, limits from y' and y, interpret, IVP
2004 AB5 B	No	Draw slope field, generalize, IVP
2005 AB6	No	Draw slope field, tangent line approximation, IVP
2005 BC4	No	Draw slope field and sketch solution on it; max/min, Euler, implicit differentiation for y'' and analyze.
2005 AB6 B	No	Draw slope field, tangent line, solve
2006 AB 5	No	Draw slope field, solve, domain
2006 BC 5	No	Implicit y", write Taylor Poly, Euler
2006 AB 5 B	No	Draw slope field, solve IVP
2006 BC5 B	No	Solve IVP, Analyze logistic DE w/o solving.