

The AP Calculus Exam

How, not only to Survive, but to Prevail...

The AP Calculus exam is the cumulation of all of the years you've spent in high school studying mathematics. It's all led up to this. The calculus you study in the last year completes the prior years of preparation. If you are reading this at the beginning of the year keep these things in mind as you go through the year. If you are reading this only a few weeks before the test think back and see how these things fit together.

Everything in calculus, and mathematics in general, is best understood verbally, numerically, analytically (that is, through the use of equations and symbols) and graphically. Look at everything from these perspectives. Look at the relationships among them — how the same idea shows up in words, in equations, in numbers and in graphs.

For example: numerically a linear function is one which when written as a table of values, regular changes in the x -values produce regular changes in the y -values. Graphically a linear function has a graph that is a straight line. Analytically it is one whose equation can be written as $y = mx + b$. And the three way are interrelated: The ratio of the changes in the table is the number m in the equation; the graph can be drawn using the number m by going up and over from one point to the next. The idea of the slope as “rise over run” expresses this verbally. Everything in mathematics and in the calculus works that way.

Learn the concepts — the exam emphasizes concepts

Learn the procedures and formulae — even though the concepts are more important than the computations you still have to do the computations. Like it or not, learn to do the algebra, the arithmetic and the graphs.

Learn to be methodical — work neatly and carefully all year.

Think about what you are doing. Watch yourself work. It is natural to concentrate on the material you know and can do, but you need to concentrate on the things you do

not (yet) know how to do. You can learn much from your mistakes. Look at a wrong answer as a green light to go in that direction until you've reached the right answer.

Reviewing for the Exam

In the few weeks before the AP Exam you will need to review what you have studied, firm up what you have learned, work on your areas of weakness and yes, memorize some formulas. You also need to prepare for the exam itself by learning what kinds of questions will be asked and how to best answer them. Specifically

- Understand the format of the exams. (See below). Know how your knowledge will be tested.
- **STUDY WHAT YOU DO NOT KNOW.** That may seem obvious but many people enjoy getting the right answers so much that they only review the stuff they know. The time to concentrate on what you know is when you are taking the test.
- Practice writing Free-Response answers. The College Board publishes copies of student answer from past years. If your teacher has some of these, look at them and learn what is expected and what is not needed.
- Plan your review carefully. Don't try to cram the weekend before the exam. The day before the test: relax, get psyched, and get a good night's sleep. The day of the test eat a good breakfast. The test is grueling, even though you're up for it. Bring a snack for the brief break between the multiple-choice and Free-response sections.

Calculators

The reason calculators are so important in learning mathematics is that they allow you do the graphical and numerical work easily, quickly and accurately. You should use your calculator all year, on homework, tests and when studying. Learn how to use it efficiently. Learn its strengths and weaknesses.

You may use your calculator any way you wish. There are four types of things you should definitely know how to do. They are

- Plot the graph of a function within an arbitrary viewing window,
- Find the zeros of functions (solve equations numerically),
- Numerically calculate the derivative of a function, and
- Numerically calculate the value of a definite integral.

You may have programs in your calculator; but you will not be asked to use them. The questions on the exam are designed so that someone with a program, or a more expensive calculator, has no advantage over someone who does not. This includes many of the built-in programs.

Be sure your calculator is set in Radian mode.

Numerical answers may be left unsimplified and in terms of π , e , etc. There is no reason to change an answer to a decimal if you don't have to. (Why take the chance on pushing the wrong button?)

Install fresh batteries before the exam.

The Format of the Exams

There are two parts to the AP exams: a multiple-choice section and a Free-response section.. The number of questions and timing may change slightly from year to year. Be sure you check the current College Board publications for your exam.

Both sections count equally towards your final grade. Both sections cover the full range of topics. It is natural to expect that different classes will cover some topics in greater detail than others; the exam will evaluate your knowledge of the calculus. It is *not* necessary to answer all the questions to get a good score. In fact the exam is made so that the average score will be about 50%, is usually a score of three.

The Current AP Calculus Exam format is

Section I Part A (55 minutes) 28 multiple-choice questions for which you may **not** use a calculator.

Section I Part B (50 minutes) 17 multiple-choice questions. You may use your calculator on this section. Some of these questions require the use of a graphing calculator others do not.

Section II Part A (45 minutes) Three Free-Response questions. You may use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may continue work on this section *without* a calculator after you start part B.

Section II Part B (45 minutes) Three Free-Response questions. You may **not** use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may use part of this time to work on Section II, Part A *without* a calculator

Multiple Choice Questions

Read each question carefully and look at the answer choices. Do the ones you are sure of. Don't struggle over one that isn't working out. Remember your time is limited and you do not need to answer all of the questions. There is a penalty for guessing, so don't guess blindly. You receive one point for each correct answer. One-quarter point is deducted for each wrong answer. Nothing is deducted for a question that is left blank. Guessing may improve your score only if you can eliminate one or more of the choices. Be sure to bubble your answer in the correct space on the answer sheet.

Types of Multiple Choice Questions

- One type of question may ask for a computation (a limit, a derivative, a definite or indefinite integral) and give five possible answers: be aware that answers which result from predictable mistakes are among the choices — work carefully, just because your answer is there doesn't mean it's correct.
- Another type may ask you only to set up a problem: looking at the answer choices may keep you from doing too much work.

- Some questions ask you to choose the one true or one false statement from a list of five statements: be sure you know if you are looking for a true or a false statement.
- Another type of question asks which of three statements is true (or false): the answer may be any one or some combination of the statements.
- Another type may ask you to choose the correct table or graph from among five choices.

Free-Response Questions

The general directions for Section II require you to show your work and indicate the methods you use to arrive at your answers. In addition, parts of questions may say, “Justify your answer” or “Show the analysis that leads to your conclusion.” Your answers will be read by calculus teachers who will judge your work. It is important that you clearly show how you arrived at your answer. Unsupported answers lose points even if the final answer is correct.

The questions are designed to show the breadth and depth of your knowledge. There are some common types of questions that are asked. There will also be questions asked in new and original ways.

Some things to keep in mind about Free-Response Questions:

- Don’t write a long essay: it's not necessary. Do show the work that you do, so that the reader will understand you. You may use common terms and names, like “the first derivative test.” You do not need to name theorems. You may show a number line as your analysis of the sign of the derivative — be sure to label it appropriately, for example y' or y'' .
- The Free-Response section of the exam rarely requires long complicated computation; if you find yourself doing a long complicated computation you’ve probably gone wrong somewhere and should start over.
- Do not explain how to do the problem you cannot do. A general explanation without work will receive no credit. You must do the problem you are given.

- *Avoid simplifying numerical answers.* Answers may be left unsimplified as fractions, radicals, powers of e , in terms of π , etc. Do not take a chance of pushing the wrong button once you have an acceptable answer. If you do arithmetic it must be done correctly. Every year students find the correct answer, change it to a decimal incorrectly and lose a point. Decimal answers (for example a definite integral on a calculator) are acceptable even if an exact answer is possible.
- If you make a mistake cross it out. Crossed out work is not read or graded. If you leave wrong work on your paper (not crossed out) it will be read and may affect your score.
- If you work the problem two different ways, choose the best one and put an X through the other. If both are left, they will both be scored and the scores will be averaged. This can lower your score even if one solution is perfect.
- Standard notation must be used. Don't use calculator notation. (For example: fnInt (x², x, 0, 2) is not acceptable, use the standard $\int_0^2 x^2 dx$.
- Answers without work do not receive full credit. Don't do work on a calculator without indicating what you are doing. For example if you are evaluating a definite integral write the integral on your paper and put the calculator answer next to it; you do not need to show the work in between (the antiderivative).
- Different calculators have different built-in utilities (for example the ability to find points of inflection, or maximum values of a function). You may have programs in your calculator to do things such as the Trapezoidal Rule. However, if you use such a built-in utility or a special program to do something other than the four things listed previously, you must show the complete set-up (the terms of the Trapezoidal Rule, the computation and analysis of the second derivative required to find a point of inflection etc.) on your paper. Only the four things listed may be done without further explanation.

- Don't put things where they are not needed. Work must be shown on the part of the answer booklet where it is used. For example, if you need a derivative in part (b) of a question and you have it in part (a) where it is not needed, you will not get credit for finding the derivative (in either part). Either copy it in part (b) or draw an arrow over to where you wrote it. You must show you know where you need the derivative as well as your ability to find it. Likewise, do not put work on the graph or drawing. It will not be read unless you specifically refer to it in the part of the answer booklet where you used it.
- Finally the parts of a Free-Response question are related to each other. This can help you in two ways:
 - Sometimes each part may be answered without reference to the other parts. Read and try of all the parts: if you cannot do part (a) maybe you can do part (b). Perhaps doing part (b) will give you a hint on how to do part (a).
 - Other times the one part will lead to the next: this is done to help you find your way through the problem. Keep in mind that this may be the case and work your way from part (a) to part (b) to part (c) even if you're not sure where the problem is heading.
- Try all of the Free-Response questions. They are written so that the first parts are easier in order to help you get started. Even if you don't get the entire problem, some points are better than no points.

Common Free-Response Mistakes

- Algebra and arithmetic mistakes.
- Missing limits of integration.
- Not considering the end points of an interval (for example, when looking for the absolute maximum value of a function).
- Giving answers from points outside the given interval.

- Not giving both coordinates of a point when required.
- Giving both coordinates when only one is asked for; remember “value of a function” means the y -value.
- Having the calculator in degrees mode.
- Not answering the question that was asked even though all the work is correct. If it is a yes or no question, say “yes” or “no.”
- Ignoring units of measure.
- Family of function problems: Questions that start with a phrase like, “This question deals with functions defined by $f(x) = 1 + b \sin(x)$ where b is a positive constant...” are meant to be done in general, *not* for a specific value of b . Even if you get the correct answer using a specific value of b , you may lose points. The reason is that, because you used a particular value, you have no way to be sure that your answers are true for all values of b .
- Don’t Curve Fit: Occasionally, a function is given as a graph or a table of values with no equation. You are being asked to demonstrate that you can work from the graphical or numerical data. The questions that follow can be answered without an equation. You may have learned to approximate functions using various curve fitting (regression) operations built into your calculator. *This should be avoided.* While this is a perfectly good approach in the real world, you may lose points because you are not working with the function you were given (only an approximation of it), and this is not one of the four allowed calculator operations.
- Using a built-in calculator utility or a program without showing all the work and justification for what you are doing. You may do only the four things you are allowed to do with a calculator on the exam.

A Word About Three-Decimal Place Accuracy.

Some answers, the evaluation of definite integrals is a prime example, must be written as decimals because they are found using a graphing calculator. These answers, and other answers that you choose to change to decimals, must be correct to three places

past the decimal point. This means that the answer may be rounded to three decimal places, truncated after the third decimal place or left with more than three decimal places as long as the first three are correct. An answer of π , which should be left as π , may be given as 3.1415926535898..., 3.142, 3.141, or even 3.14199999. If the number ends in zeros, they may be omitted; thus 17.320 may be given as 17.32 and 56.000 may be given as 56.

Too often, students may choose to give decimal answers when they are not required. Once a Free-Response answer is entirely in terms of numbers there is no need to change the number to a decimal. For example, 1999 AB 1(c) does *not* require a decimal answer: $-\frac{1}{2}\cos 4 + \frac{7}{2}$ is sufficient. If the decimal is correct (to three decimal places) the student will receive the credit. However, if you change a correct answer to an incorrect decimal (including one with too few decimals) then you will lose credit. The moral is: avoid arithmetic, avoid decimals; give them only if you cannot give anything else.

Rounding too soon is another common mistake made by students. Computations should be done with more decimal places than is required in the final answer. Learn how to store the intermediate values in your calculator and recall them when you need them in a computation. If premature rounding affects the three decimal place accuracy of the final answer, you will not be given the answer point. However, a rounded answer used in the next part of a problem will not be held against you.

Good Luck!