AP Calculus Explorations for Teachers

By Lin McMullin

PARTICLE MOTION ON A LINE.

The AP Calculus Course Description indicates that AB and BC calculus students should understand

- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.
- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- The derivative interpreted as the instantaneous rate of change
- Equations involving derivatives
- The distance traveled by a particle along a number line

These questions may give the position equation, or the velocity equation or the acceleration equation along with initial conditions. Students may be asked about the motion of the particle: its direction, when it changes direction, how far it goes in one direction, etc. Speed, the absolute value of velocity, is also a common topic. The relationship between speed, and velocity and acceleration is also tested.

The particle may be a "particle," a person, car, etc. The position, velocity, or acceleration may be given as an equation, a graph, or a table. There are a lot of different things students may be asked to find. While particles don't really move in this way, the question is a versatile way to test a variety of calculus concepts.

Most textbooks deal only with uniformly accelerated motion (projectiles and falling objects); which are only a subset of motion problems. Therefore, you will need to supplement what's in your textbook.

What students should know how to do (AB and BC):

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position and / or velocity.
- Distinguish between position at some time (displacement) and the distance traveled during the time.
    - The total distance traveled is the definite integral of the absolute value of the rate of change (velocity): $\int_a^b \left|v(t)\right| dt$.
    - The net distance (displacement) is the definite integral of the rate of change (velocity): $\int_a^b v(t)\,dt$.
    - The final position is the initial position plus the definite integral above.
    - The position at any time t is given by the accumulation function $s(t) = s(a) + \int_a^t v(x)\,dx$.

- Find the speed at a particular time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration.
- Determine if the speed is increasing or decreasing.
    - If at some time, the velocity and acceleration have the *same* sign then the speed is increasing.
    - If they have *different* signs the speed is decreasing.
- Use difference quotient to approximate derivative.
- Riemann sum approximations.
- Units of measure.
- Interpret meaning of integral in context of the problem.

1. Exercise 1: 2002 AB 3 (Graphing calculators allowed)

The stem:
Given a velocity equation with domain and an initial condition. (Notice the $\frac{\pi}{3}$.)

- What can you determine about the motion from the given information?

The parts:

- Part (a) asks for the acceleration at a particular time.
    - There were at least 3 different correct forms not counting the decimal answer. How many can you find?
    - Give a decimal answer.
- Part (b) requires some reading and understanding
    - What idea or concept is likely to trip students up in this part?
    - How do you explain this concept to students?
    - What's the "bottom line" for determining whether the speed is increasing or decreasing?

- Part (c) asks for the total distance traveled for $t \in [0, 4]$
    - How many ways can you find this answer? What are they?

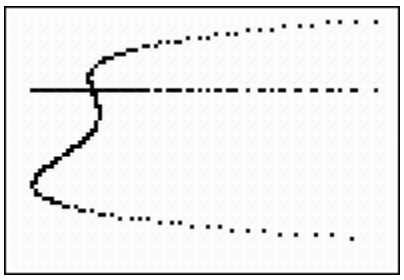
- Part (d) asks for the position at $t = 4$
    - Again there are several approaches to answering this part. What are they?

Discuss the scoring standard for this question.

2. Seeing Motion on a line A calculator demonstration.[1]

In the parametric graphing mode, turn the axes "off" and set you calculator to draw "Dot," and use these Window settings:

tmin = –4, tmax = 5, tscl = 0.05,
xmin = –3, xmax = 10, sscl = 1,
ymin = –5, ymax = 5 and yscl = 1.



Enter this parametric equation: $xt1 = 0.05(t - 1)^2 (t^2 - 9)$ $yt1 = 2$. Graph to see the particle move on a number line

Make: $xt2 = 0.05(t - 1)^2 (t^2 - 9)$ $yt1 = t$ and re-graph to see the particle move as a function of time. Graph both simultaneously to better understand what's happening.

3. Exercise 2 1998 AB 3: (Graphing calculator allowed)

The stem:
The information about the motion of a car (aka a particle) is given in both a table and a graph.

- What information can you read or assume from the graph that you cannot assume from the table? Why?

[1] From *Teaching AP Calculus* © 2005 by Lin McMullin. p. 152 – 153.

- Would you advise students to enter the data from the table into their statistic editor and find a regression equation? Why or why not?

The parts:

- Part (a): How do you tell from a *velocity* graph when the *acceleration* is positive? Explain?

- Part (a): How do you tell from a *velocity* table when the *acceleration* is positive? Explain? Are you making any unwarranted assumptions?

- Part (b): What relationship are you using to find average acceleration?

- Part(c): Here you are asked to approximate the average acceleration.
  - Can you find the actual acceleration at $t = 40$? Why or why not?

  - There are several ways to do this problem. Name at least 3.

- Part (d): The Riemann sum is a pretty straightforward computation.
  - There are three important things to consider in the explanation. An explanation of the meaning of a definition in the context of the problem is a common writing question that students should be ready for. What 3 things need to be included in a complete answer?

Discuss the scoring standard for this question.

4. Exercise 3: Now you're on your own: Do 2003 AB 2. Timed: 15 minutes.

Discuss the scoring standard for this question.

5. Exercise 4: Many of the skills needed to answer these questions are taught in the 4-years of mathematics prior to studying the calculus. This exercise will give you some practice identifying these skills and show you how to start preparing your students for AP Calculus long before they start the class.

Return to 1998 AB 3 and write 2 questions based on this questions or its graph that would be suitable to use at each level listed below. You may, if necessary, revise the stem or graph slightly.

Algebra 1

Geometry

Algebra 2

Trigonometry

Precalculus

6. Brainstorm:
What can you have your STUDENTS doing on a regular basis so that they will master these concepts?

1.

2.

3.

4.

5.

Index to AP Free-response questions Topic 2.

The questions themselves can be found at AP Central. Click on "The Exams" and then on "Exam Questions" then "Calculus AB" or "Calculus BC" in the table. or go to http://apcentral.collegeboard.com/apc/members/exam/exam_questions/1997.html

Topic 2: Particle Motion on a line for AB and BC		
Year & Question	**GCA**	**Comments**
1998 AB3	Yes	Given graph & table: difference quotient, Riemann sum
1999 AB 1	Yes	Given v: equation analyze motion, find a, s and total distance
2000 AB2-BC2	Yes	Given graph & equation
2001 AB3-BC3	Yes	Given *a* graph: analyze *v*, max/min, FTC
2002 AB3	Yes	Given *v* equation: Speed/velocity, Speed increasing?
2002 AB3 B	Yes	Given *v* equation: Sketch graph, analyze motion
2003 AB2	Yes	Given *v* graph: Speed increasing? Analyze motion
2003 AB4 B	No	Given *v* graph: Speed increasing? Analyze motion
2004 AB3	Yes	Given *v* graph: Speed increasing? Analyze motion
2004 AB3-BC3 B	Yes	Given table of *v*: explain integral, MVT, average *v*
2005 AB5-BC5	No	Given *v* graph: distance, *a*, average rate of change
2005 AB3 B	Yes	Given *v* equation: analyze motion.
2006 AB4	No	Table & equation, average, Riemann sum,
2006 AB6 B	No	From table, distance, FTC, analyze motion

For similar explorations of other AP Calculus free-response type problems see

http://www.linmcmullin.net/AP_Calculus_NEW.html or go to

www.LinMcMullin.net > RESOURCES > AP* CALCULUS