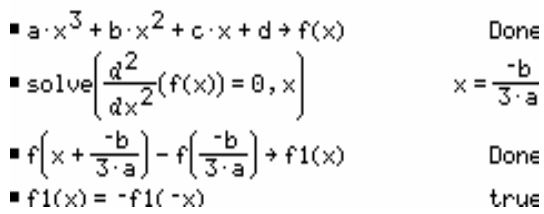
Problems for CAS Solution

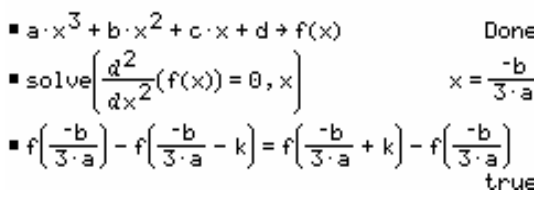
Presented by Lin McMullin

www.linmcmullin.net

Solutions

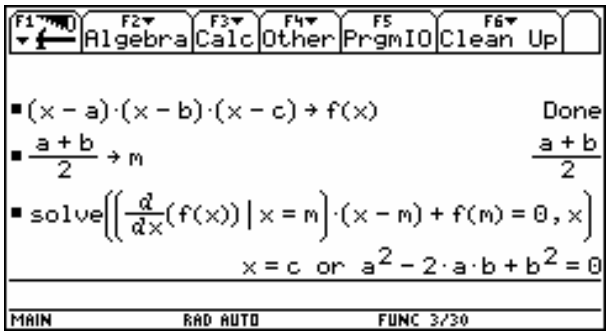
1. Two solutions:



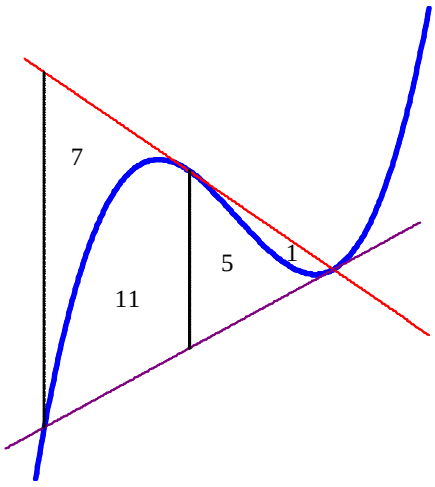


2. Beside what we were expecting, what does the last line tell us?

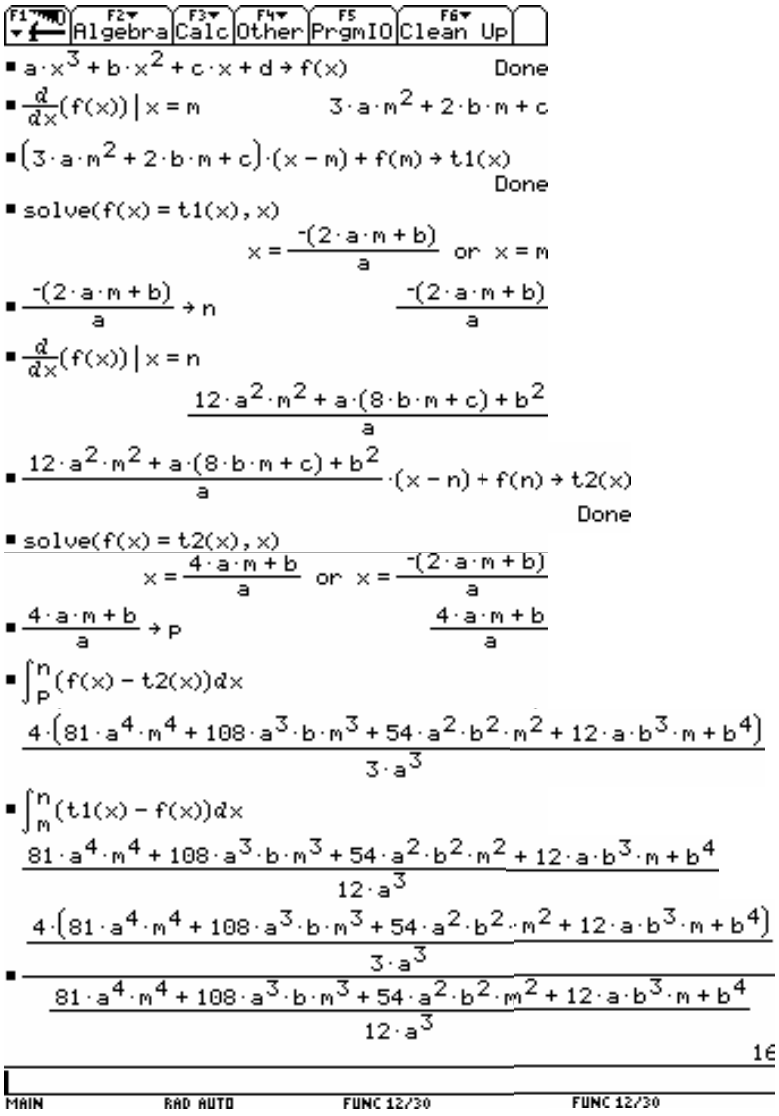
When $a = b$ the cubic is tangent to the x-axis; thex-axis becomes the t angent line, and therefore will contain the third root at c.



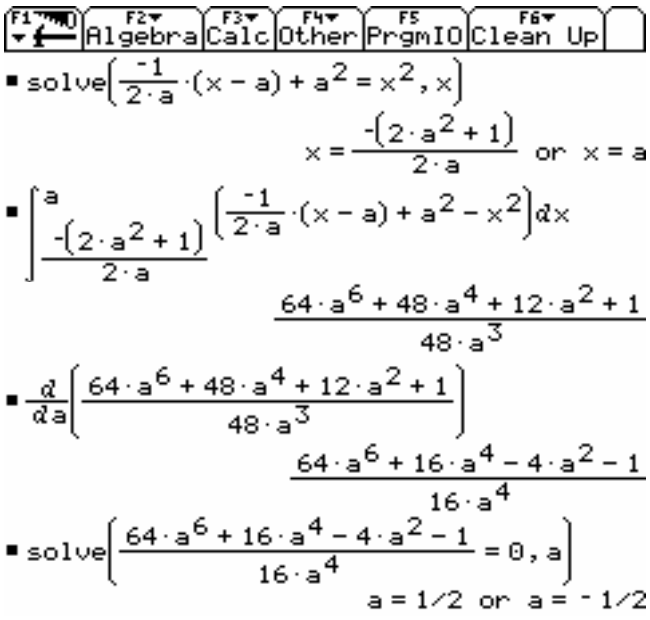
3. a. 16:1; b. (next page)
Some other ratios:



Suggested by Audrey Weeks in *Algebra in Motion* (www.calculusinmotion.com)



4. $x = ½$



5. See http://www.linmcmullin.net/Golden_Ratio_in_Quartics.pdf

6.

Define $\text{slope}(a1,b1,a2,b2)=\frac{b2-b1}{a2-a1}$ — Done

slope(-5,2,11.3,7.1)	51/163
slope(11.3,7.1,16.4,5)	-7/17
slope(16.4,5,.1,-.1)	51/163
slope(.1,-.1,-5,2)	-7/17

slope(-5,2,16.4,5)→md1	15/107
slope(11.3,7.1,.1,-.1)→md2	9/14
md1*md2=-1	false

$\left\{\frac{a1+a2}{2} \quad \frac{b1+b2}{2}\right\} \rightarrow \text{midpt}(a1,b1,a2,b2)$ — Done

midpt(-5,2,16.4,5)	{57/10 7/2}
midpt(11.3,7.1,.1,-.1)	{57/10 7/2}

7.

$\sqrt{(a2-a1)^2+(b2-b1)^2} \rightarrow \text{dist}(a1,b1,a2,b2)$ — Done

dist(a,y,0,0)	$\sqrt{y^2+a^2}$
dist(a,y,2·a,0)	$\sqrt{y^2+a^2}$

8.

$\sqrt{(a2-a1)^2+(b2-b1)^2} \rightarrow \text{dist}(a1,b1,a2,b2)$ — Done

dist(-3,2,5,4) $2\cdot\sqrt{17}$

solve(dist(x,y,-3,2)=dist(x,y,5,4),y)

$y=-(4\cdot x-7)$

solve(dist(x,y,-3,2)+dist(x,y,5,4)=9,y)

$y=\frac{9\cdot\sqrt{-13\cdot(4\cdot x^2-8\cdot x-73)}+2\cdot(16\cdot x+215)}{154}$ or $y=\frac{-\left(9\cdot\sqrt{-13\cdot(4\cdot x^2-8\cdot x-73)}-2\cdot(16\cdot x+215)\right)}{154}$

These are the equations of two halves of an ellipse. Try graphing them.

9. SSS

$$\text{solve}(\text{dist}(4.5\cdot\cos(\theta), 4.5\cdot\sin(\theta), 8, 0) = 6, \theta) \mid \theta > 0 \text{ and } \theta < 180$$
$$\theta = 47.9222147823$$

10. SSA: From the top: 2 solutions, no solutions, one solution (the negative answer is extraneous)

$$\text{solve}(\text{dist}(8.75\cdot\cos(37.8), 8.75\cdot\sin(37.8), c, 0) = 6, c)$$
$$c = 4.22333415191 \text{ or } c = 9.60437856467$$
$$\text{solve}(\text{dist}(8.75\cdot\cos(37.8), 8.75\cdot\sin(37.8), c, 0) = 2, c)$$
$$\text{false}$$
$$\text{solve}(\text{dist}(8.75\cdot\cos(37.8), 8.75\cdot\sin(37.8), c, 0) = 10, c)$$
$$c = -1.52645819409 \text{ or } c = 15.3541709107$$

11. ASA: Solved as a system of 2 equations in *a* and *b*.

$$\text{solve}(a\cdot\cos(43.5) + b\cdot\cos(50.7) = 15 \text{ and } a\cdot\sin(43.5) = b\cdot\sin(50.7), \{a \quad b\})$$
$$a = 11.6388595683 \text{ and } b = 10.3531221808$$

12.

$$\text{solve}\left(\frac{1}{h} - 6 = 12 - \frac{1}{h}, h\right) \qquad h = 1/9$$
$$\text{solve}\left(\frac{1}{h} - \frac{1}{a} = \frac{1}{b} - \frac{1}{h}, h\right) \qquad h = \frac{2\cdot a\cdot b}{a+b}$$
$$\frac{1}{h} - \frac{1}{a} \mid h = \frac{2\cdot a\cdot b}{a+b} \qquad \frac{a-b}{2\cdot a\cdot b}$$
$$\frac{1}{b} - \frac{1}{h} \mid h = \frac{2\cdot a\cdot b}{a+b} \qquad \frac{a-b}{2\cdot a\cdot b}$$

$$\text{comDenom}\left(\frac{1}{a} + (n-1)\cdot\left(\frac{1}{\frac{2\cdot a\cdot b}{a+b}} - \frac{1}{a}\right)\right) \mid n = \{1 \quad 2 \quad 3\}$$
$$\left\{\frac{1}{a} \quad \frac{a+b}{2\cdot a\cdot b} \quad \frac{1}{b}\right\}$$

13.

$$\text{solve}\left(x + \frac{1}{x} = 17/5, x\right)$$
$$x = \frac{3\cdot\sqrt{21} + 17}{10} \text{ or } x = \frac{-(3\cdot\sqrt{21} - 17)}{10}$$
$$x + \frac{1}{x} \mid x = \left\{\frac{3\cdot\sqrt{21} + 17}{10} \quad \frac{-(3\cdot\sqrt{21} - 17)}{10}\right\}$$
$$\{17/5 \quad 17/5\}$$

14.

- seq(n,n,3,15,3) {3 6 9 12 15}
- seq(3·n,n,1,5) {3 6 9 12 15}
- seq(n+3,n,0,12,3) {3 6 9 12 15}
- $\text{seq}\left(\left(\sqrt{3\cdot n}\right)^2, n, 1, 5\right)$ {3 6 9 12 15}
- seq(20−n,n,17,5,−3) {3 6 9 12 15}

seq(n*cos(2nπ),n,3,15,3)

15.

- $\frac{x^3-9\cdot x^2+14\cdot x}{-x^2-3\cdot x+10}$ $\frac{-x\cdot(x-7)}{x+5}$
- factor$(x^3-9\cdot x^2+14\cdot x)$ $x\cdot(x-7)\cdot(x-2)$
- factor$(-x^2-3\cdot x+10)$ $-(x-2)\cdot(x+5)$

16. There's the Golden Ratio again!

- factor(x^5-1,x)

$$(x-1)\cdot\left(x^2-\frac{(\sqrt{5}-1)\cdot x}{2}+1\right)\cdot\left(x^2+\frac{(\sqrt{5}+1)\cdot x}{2}+1\right)$$

factor(x^5−1,x)

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