

Problems for CAS Solution

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Solutions

1. Two solutions:

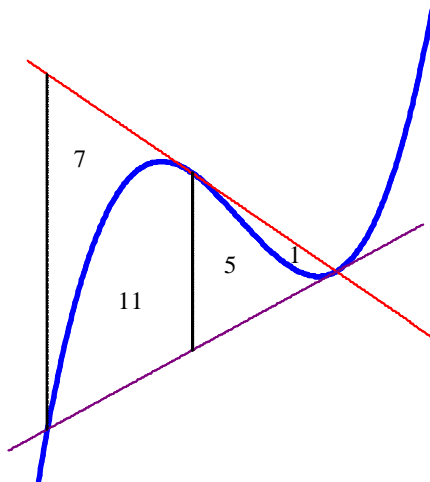
$\blacksquare a \cdot x^3 + b \cdot x^2 + c \cdot x + d \rightarrow f(x)$	Done	$\blacksquare a \cdot x^3 + b \cdot x^2 + c \cdot x + d \rightarrow f(x)$	Done
$\blacksquare \text{solve}\left(\frac{d^2}{dx^2}(f(x)) = 0, x\right)$	$x = \frac{-b}{3 \cdot a}$	$\blacksquare \text{solve}\left(\frac{d^2}{dx^2}(f(x)) = 0, x\right)$	$x = \frac{-b}{3 \cdot a}$
$\blacksquare f\left(x + \frac{-b}{3 \cdot a}\right) - f\left(\frac{-b}{3 \cdot a}\right) \rightarrow f1(x)$	Done	$\blacksquare f\left(\frac{-b}{3 \cdot a}\right) - f\left(\frac{-b}{3 \cdot a} - k\right) = f\left(\frac{-b}{3 \cdot a} + k\right) - f\left(\frac{-b}{3 \cdot a}\right)$	true
$\blacksquare f1(x) = -f1(-x)$	true		

2. Beside what we were expecting, what does the last line tell us?

When $a = b$ the cubic is tangent to the x -axis; the x -axis becomes the tangent line, and therefore will contain the third root at c .

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\blacksquare (x - a) \cdot (x - b) \cdot (x - c) \rightarrow f(x)$					Done
$\blacksquare \frac{a + b}{2} \rightarrow m$					$\frac{a + b}{2}$
$\blacksquare \text{solve}\left(\left[\frac{d}{dx}(f(x)) \mid x = m\right] \cdot (x - m) + f(m) = 0, x\right)$					
$x = c \text{ or } a^2 - 2 \cdot a \cdot b + b^2 = 0$					
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3. a. 16:1; b. (next page)
Some other ratios:



Suggested by Audrey Weeks in *Algebra in Motion* (www.calculusinmotion.com)

	F1 Algebra	F2 Calc	F3 Other	F4 PrgmIO	F5 Clean Up	
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- 1 $a \cdot x^3 + b \cdot x^2 + c \cdot x + d + f(x)$ Done
- 2 $\frac{d}{dx}(f(x)) | x = m$ $3 \cdot a \cdot m^2 + 2 \cdot b \cdot m + c$
- 3 $(3 \cdot a \cdot m^2 + 2 \cdot b \cdot m + c) \cdot (x - m) + f(m) \rightarrow t1(x)$ Done
- 4 $\text{solve}(f(x) = t1(x), x)$
 $x = \frac{-(2 \cdot a \cdot m + b)}{a}$ or $x = m$
- 5 $\frac{-(2 \cdot a \cdot m + b)}{a} \rightarrow n$ $\frac{-(2 \cdot a \cdot m + b)}{a}$
- 6 $\frac{d}{dx}(f(x)) | x = n$
 $\frac{12 \cdot a^2 \cdot m^2 + a \cdot (8 \cdot b \cdot m + c) + b^2}{a}$
- 7 $\frac{12 \cdot a^2 \cdot m^2 + a \cdot (8 \cdot b \cdot m + c) + b^2}{a} \cdot (x - n) + f(n) \rightarrow t2(x)$ Done
- 8 $\text{solve}(f(x) = t2(x), x)$
 $x = \frac{4 \cdot a \cdot m + b}{a}$ or $x = \frac{-(2 \cdot a \cdot m + b)}{a}$
- 9 $\frac{4 \cdot a \cdot m + b}{a} \rightarrow p$ $\frac{4 \cdot a \cdot m + b}{a}$
- 10 $\int_p^n (f(x) - t2(x)) dx$
 $\frac{4 \cdot (81 \cdot a^4 \cdot m^4 + 108 \cdot a^3 \cdot b \cdot m^3 + 54 \cdot a^2 \cdot b^2 \cdot m^2 + 12 \cdot a \cdot b^3 \cdot m + b^4)}{3 \cdot a^3}$
- 11 $\int_m^n (t1(x) - f(x)) dx$
 $\frac{81 \cdot a^4 \cdot m^4 + 108 \cdot a^3 \cdot b \cdot m^3 + 54 \cdot a^2 \cdot b^2 \cdot m^2 + 12 \cdot a \cdot b^3 \cdot m + b^4}{12 \cdot a^3}$
 $\frac{4 \cdot (81 \cdot a^4 \cdot m^4 + 108 \cdot a^3 \cdot b \cdot m^3 + 54 \cdot a^2 \cdot b^2 \cdot m^2 + 12 \cdot a \cdot b^3 \cdot m + b^4)}{3 \cdot a^3}$
 $\frac{81 \cdot a^4 \cdot m^4 + 108 \cdot a^3 \cdot b \cdot m^3 + 54 \cdot a^2 \cdot b^2 \cdot m^2 + 12 \cdot a \cdot b^3 \cdot m + b^4}{12 \cdot a^3}$

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4. $x = 1/2$

	F1 Algebra	F2 Calc	F3 Other	F4 PrgmIO	F5 Clean Up	
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- 1 $\text{solve}\left(\frac{-1}{2 \cdot a} \cdot (x - a) + a^2 = x^2, x\right)$
 $x = \frac{-(2 \cdot a^2 + 1)}{2 \cdot a}$ or $x = a$
- 2 $\int_a^a \frac{-(2 \cdot a^2 + 1)}{2 \cdot a} \left(\frac{-1}{2 \cdot a} \cdot (x - a) + a^2 - x^2\right) dx$
 $\frac{64 \cdot a^6 + 48 \cdot a^4 + 12 \cdot a^2 + 1}{48 \cdot a^3}$
- 3 $\frac{d}{da} \left(\frac{64 \cdot a^6 + 48 \cdot a^4 + 12 \cdot a^2 + 1}{48 \cdot a^3} \right)$
 $\frac{64 \cdot a^6 + 16 \cdot a^4 - 4 \cdot a^2 - 1}{16 \cdot a^4}$
- 4 $\text{solve}\left(\frac{64 \cdot a^6 + 16 \cdot a^4 - 4 \cdot a^2 - 1}{16 \cdot a^4} = 0, a\right)$
 $a = 1/2$ or $a = -1/2$

5. See http://www.linmcmullin.net/Golden_Ratio_in_Quartics.pdf

6.
 ■ Define $\text{slope}(a1, b1, a2, b2) = \frac{b2 - b1}{a2 - a1}$ Done
 ■ $\text{slope}(-5, 2, 11.3, 7.1)$ 51/163
 ■ $\text{slope}(11.3, 7.1, 16.4, 5)$ -7/17
 ■ $\text{slope}(16.4, 5, .1, -.1)$ 51/163
 ■ $\text{slope}(.1, -.1, -5, 2)$ -7/17

■ $\text{slope}(-5, 2, 16.4, 5) \rightarrow \text{md1}$ 15/107
 ■ $\text{slope}(11.3, 7.1, .1, -.1) \rightarrow \text{md2}$ 9/14
 ■ $\text{md1} * \text{md2} = -1$ false

■ $\left\{ \frac{a1 + a2}{2} \quad \frac{b1 + b2}{2} \right\} \rightarrow \text{midpt}(a1, b1, a2, b2)$ Done
 ■ $\text{midpt}(-5, 2, 16.4, 5)$ (57/10 7/2)
 ■ $\text{midpt}(11.3, 7.1, .1, -.1)$ (57/10 7/2)

7.
 ■ $\sqrt{(a2 - a1)^2 + (b2 - b1)^2} \rightarrow \text{dist}(a1, b1, a2, b2)$ Done
 ■ $\text{dist}(a, y, 0, 0)$ $\sqrt{y^2 + a^2}$
 ■ $\text{dist}(a, y, 2 \cdot a, 0)$ $\sqrt{y^2 + a^2}$

8.
 ■ $\sqrt{(a2 - a1)^2 + (b2 - b1)^2} \rightarrow \text{dist}(a1, b1, a2, b2)$ Done
 ■ $\text{dist}(-3, 2, 5, 4)$ $2 \cdot \sqrt{17}$
 ■ $\text{solve}(\text{dist}(x, y, -3, 2) = \text{dist}(x, y, 5, 4), y)$
 $y = -(4 \cdot x - 7)$

■ $\text{solve}(\text{dist}(x, y, -3, 2) + \text{dist}(x, y, 5, 4) = 9, y)$
 $y = \frac{9 \cdot \sqrt{-13 \cdot (4 \cdot x^2 - 8 \cdot x - 73)} + 2 \cdot (16 \cdot x + 215)}{154}$ or $y = \frac{-\left(9 \cdot \sqrt{-13 \cdot (4 \cdot x^2 - 8 \cdot x - 73)} - 2 \cdot (16 \cdot x + 215)\right)}{154}$

These are the equations of two halves of an ellipse. Try graphing them.

9. SSS

$$\blacksquare \text{solve}(\text{dist}(4.5 \cdot \cos(\theta), 4.5 \cdot \sin(\theta), 8, 0) = 6, \theta) \mid \theta > 0 \text{ and } \theta < 180$$

$$\theta = 47.9222147823$$

10. SSA: From the top: 2 solutions, no solutions, one solution (the negative answer is extraneous)

$$\blacksquare \text{solve}(\text{dist}(8.75 \cdot \cos(37.8), 8.75 \cdot \sin(37.8), c, 0) = 6, c)$$

$$c = 4.22333415191 \text{ or } c = 9.60437856467$$

$$\blacksquare \text{solve}(\text{dist}(8.75 \cdot \cos(37.8), 8.75 \cdot \sin(37.8), c, 0) = 2, c)$$

$$\text{false}$$

$$\blacksquare \text{solve}(\text{dist}(8.75 \cdot \cos(37.8), 8.75 \cdot \sin(37.8), c, 0) = 10, c)$$

$$c = -1.52645819409 \text{ or } c = 15.3541709107$$

11. ASA: Solved as a system of 2 equations in a and b .

$$\blacksquare \text{solve}(a \cdot \cos(43.5) + b \cdot \cos(50.7) = 15 \text{ and } a \cdot \sin(43.5) = b \cdot \sin(50.7), \{a \ b\})$$

$$a = 11.6388595683 \text{ and } b = 10.3531221808$$

12.

$$\blacksquare \text{solve}\left(\frac{1}{h} - 6 = 12 - \frac{1}{h}, h\right) \quad h = 1/5$$

$$\blacksquare \text{solve}\left(\frac{1}{h} - \frac{1}{a} = \frac{1}{b} - \frac{1}{h}, h\right) \quad h = \frac{2 \cdot a \cdot b}{a + b}$$

$$\blacksquare \frac{1}{h} - \frac{1}{a} \mid h = \frac{2 \cdot a \cdot b}{a + b} \quad \frac{a - b}{2 \cdot a \cdot b}$$

$$\blacksquare \frac{1}{b} - \frac{1}{h} \mid h = \frac{2 \cdot a \cdot b}{a + b} \quad \frac{a - b}{2 \cdot a \cdot b}$$

$$\blacksquare \text{comDenom}\left(\frac{1}{a} + (n - 1) \cdot \left(\frac{1}{\frac{2 \cdot a \cdot b}{a + b}} - \frac{1}{a}\right)\right) \mid n = \{1 \ 2 \ 3\}$$

$$\left\{ \frac{1}{a} \quad \frac{a + b}{2 \cdot a \cdot b} \quad \frac{1}{b} \right\}$$

13.

$$\blacksquare \text{solve}\left(x + \frac{1}{x} = 17/5, x\right)$$

$$x = \frac{3 \cdot \sqrt{21} + 17}{10} \text{ or } x = \frac{-(3 \cdot \sqrt{21} - 17)}{10}$$

$$\blacksquare x + \frac{1}{x} \mid x = \left\{ \frac{3 \cdot \sqrt{21} + 17}{10} \quad \frac{-(3 \cdot \sqrt{21} - 17)}{10} \right\}$$

$$\{17/5 \ 17/5\}$$

- 14.
- `seq(n, n, 3, 15, 3)` `{3 6 9 12 15}`
 - `seq(3·n, n, 1, 5)` `{3 6 9 12 15}`
 - `seq(n + 3, n, 0, 12, 3)` `{3 6 9 12 15}`
 - `seq((√3·n)2, n, 1, 5)` `{3 6 9 12 15}`
 - `seq(20 - n, n, 17, 5, -3)` `{3 6 9 12 15}`
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- `seq(n*cos(2nπ), n, 3, 15, 3)`**

- 15.
- $\frac{x^3 - 9x^2 + 14x}{-x^2 - 3x + 10}$ $\frac{-x \cdot (x - 7)}{x + 5}$
 - `factor(x3 - 9·x2 + 14·x)` `x·(x - 7)·(x - 2)`
 - `factor(-x2 - 3·x + 10)` `-(x - 2)·(x + 5)`

16. There's the Golden Ratio again!

- `factor(x5 - 1, x)`

$$(x - 1) \cdot \left(x^2 - \frac{(\sqrt{5} - 1) \cdot x}{2} + 1 \right) \cdot \left(x^2 + \frac{(\sqrt{5} + 1) \cdot x}{2} + 1 \right)$$

`factor(x5 - 1, x)`

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