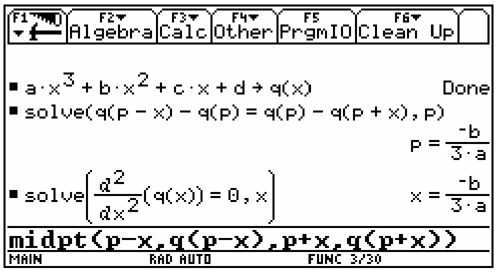
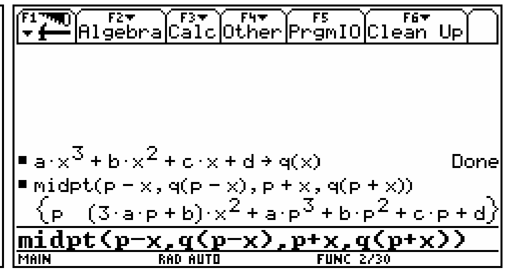
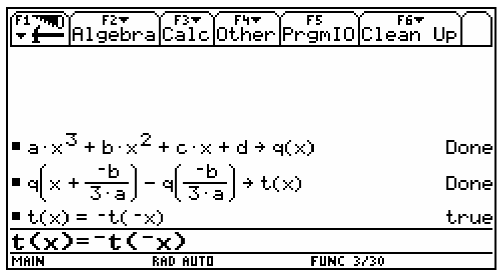
Problems for CAS Solution
Presented by
Lin McMullin

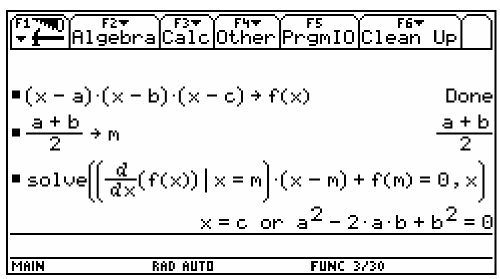
1. Prove that the graph of every cubic polynomial has a point of symmetry (or the graph is symmetric to its point of inflection).





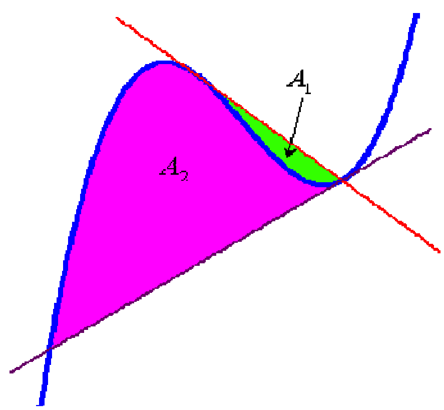


2. Prove that the tangent line drawn to a cubic polynomial at the point where x = average of two of its roots, intersects the polynomial on the x-axis at the third root.

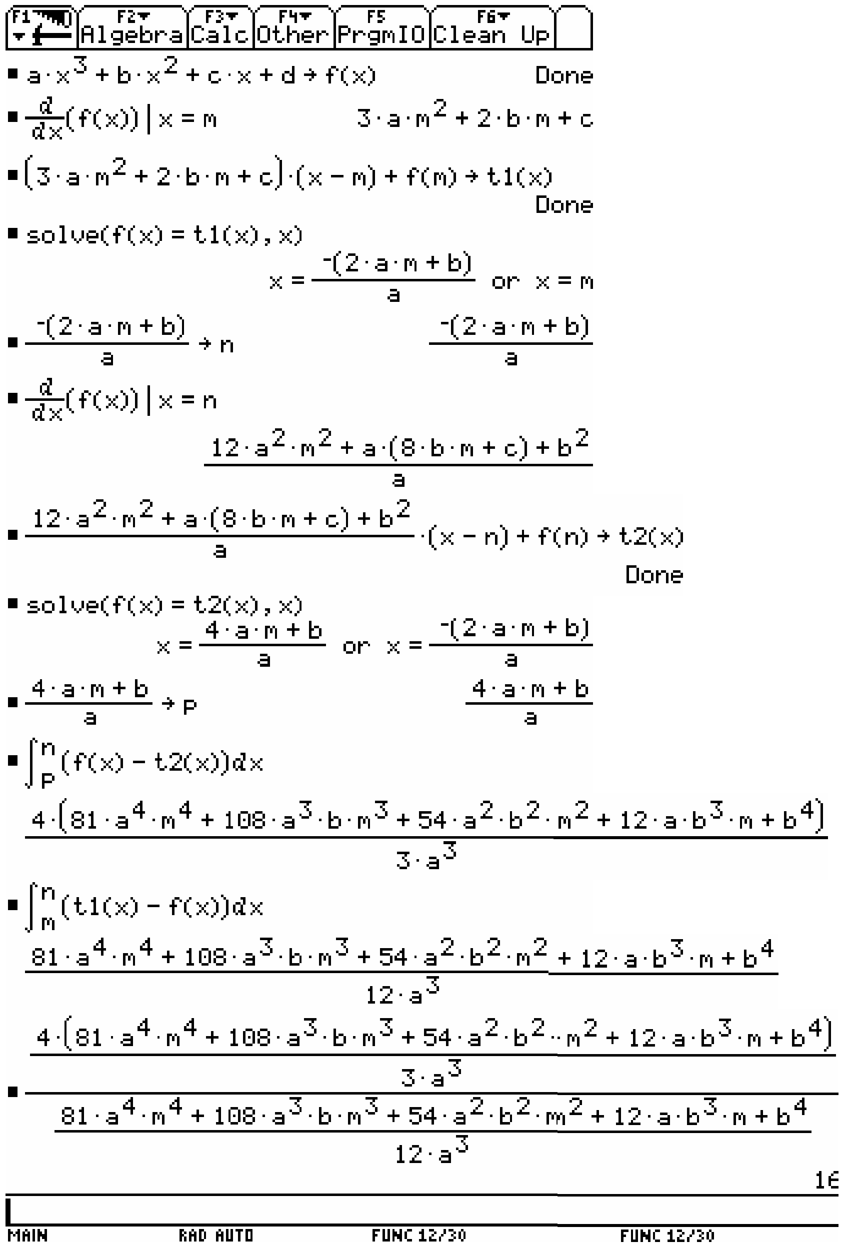


Beside what we were expecting, what does the last line tell us?

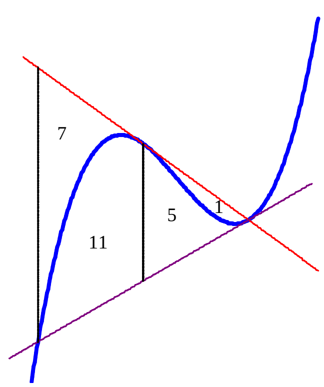
3. Draw a tangent line at any point, other than the point of inflection of a cubic polynomial. This tangent will intersect the cubic at a second point; draw a tangent line at this second point. The second tangent will intersect the cubic at a third point. Let A_1 be the area of the region between the first tangent line and the cubic and let A_2 be the area of the region between the cubic and the second tangent line. A general graph is given below. The interesting result is that the ratio $A_2 : A_1$ is constant.
(A) Find the ratio $A_2 : A_1$.
(B) Prove that the ratio is constant.



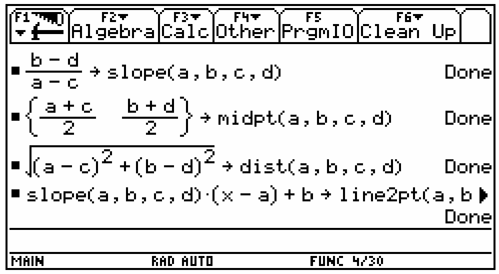
Suggested by *Algebra in Motion* by Audrey Weeks at www.calculusinmotion.com
(A) 16:1;   (B): CAS solution:



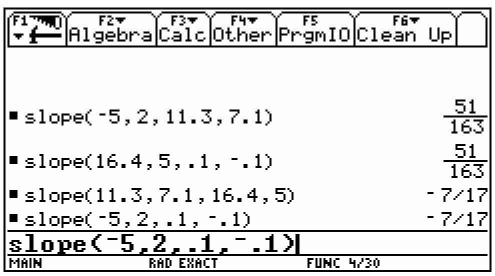
And some other Ratios:

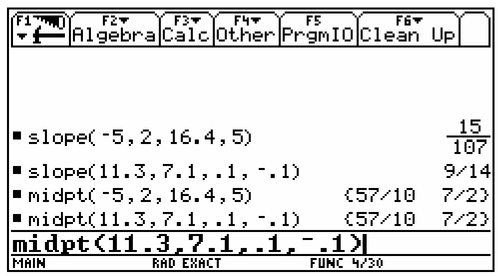


Analytic Geometry:

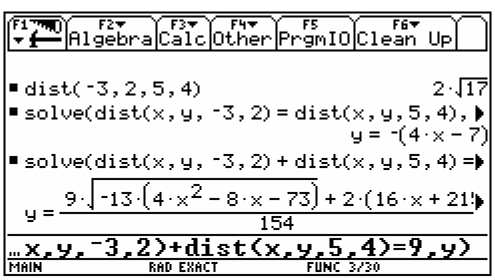


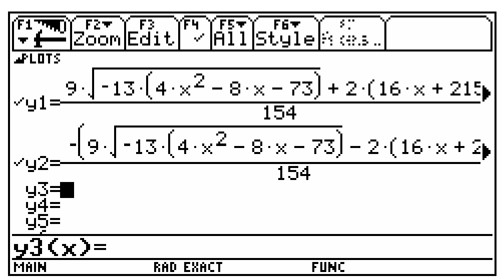
4. Given the quadrilateral with vertices
   $A(-5,2)$, $B(11.3,7.1)$, $C(16.4,5.0)$ and $D(0.1,-0.1)$
   (a) Show that *ABCD* is a parallelogram.
   (b) Are the diagonals perpendicular? Show how you know.
   (c) Show that the diagonals bisect each other.

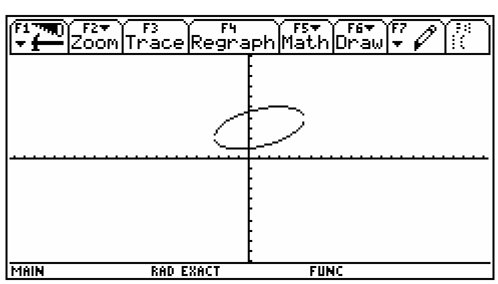




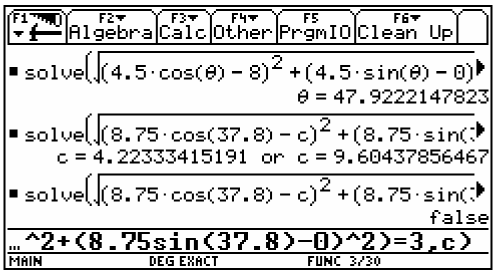
5. Given the points $A(-3,2)$ and $B(5,4)$
   a. Find the length *AB*.
   b. Write an equation of the perpendicular bisector of $\overline{AB}$.
   c. Write an equation of the set of points (*x*, *y*) such that the sum of the distances from (*x*, *y*) to *A* and *B* is 9.
   d. Graph the locus found in part (c).

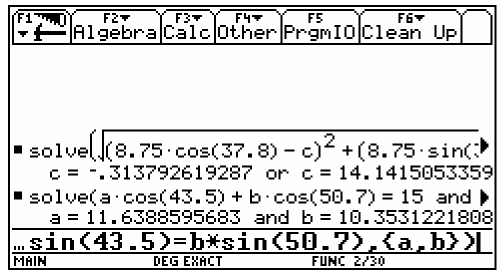






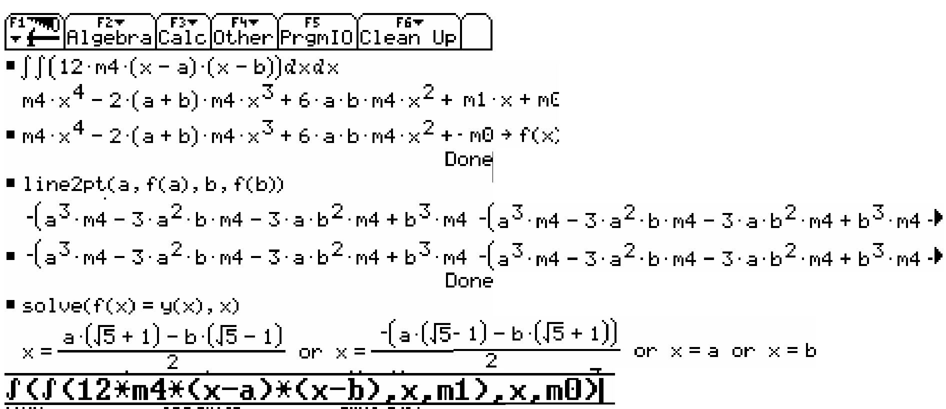
6. Trigonometry.
    a. SSS: A triangle has sides of 4.5, 6 and 8. Find the measure of the angle opposite the side of 6.
    b. SSA: In triangle ABC, angle A = 37.8°, side b = 8.75 and side a = 6. Find the measure of length of side AB = c.
    c. SSA: In triangle ABC, angle A = 37.8°, side b = 8.75 and side a = 3. Find the measure of length of side AB = c.
    d. SSA: In triangle ABC, angle A = 37.8°, side b = 8.75 and side a = 9. Find the measure of length of side AB = c.
    e. ASA : In triangle ABC, angle A = 50.7°, angle B = 43.5° and AB = 15. Find the lengths of the other 2 sides.





7. Where else does the line through the points of inflection of a 4th degree polynomial intersect the polynomial?

    (Note: First line's entry format is shown at the bottom; this shows how to enter, + C, the constant of integration, here first m_1 then m_0,)



Why is the solution so unexpected?

8. How is doing math with a CAS different than do math without a CAS?

- CAS removes the necessity to do algemetic, so we can concentrate on the mathematics.
- Knowing how to use the CAS allows you to improve the CAS by adding routines you need to do standard problems the "usual" way.
- New approaches for doing problems appear once you stop worrying about the algemetic.
- 'Go for the equation' – since the CAS can solve (in closed form or not) almost any equation, you needn't worry about how difficult it is to solve.
- A CAS will simplify just about any expression so we don't have to avoid complicated expressions – in fact we can complicate what we have, if that makes the flow of work easier.
- One still needs to know the mathematics to do the set-up and to interpret the answers.

9. What are the implications for teaching?

   Good CAS use is a new skill, not just a new tool that students must be taught and encouraged to learn.

   To do this we will need

- A willingness to accept new ways of doing problems
- A willingness to accept showing a different kind of work
- A change in the meaning of "simplify."
- A good source of (better) problems for students to attempt.