Problems for CAS Solution

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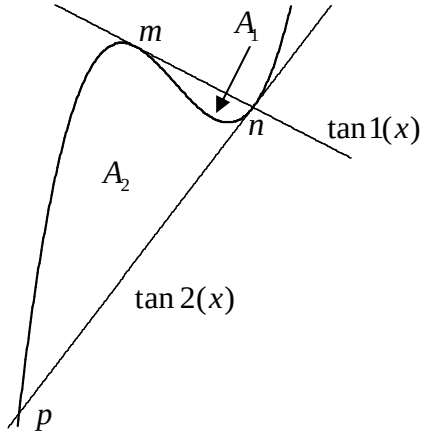
New Problems

1. Prove that the graph of every cubic polynomial is symmetric about its point of inflection.
2. Prove that the tangent line drawn to a cubic polynomial at the point where x = average of two of its roots, intersects the polynomial on the x-axis at the third root.
3. Draw a tangent line at any point, m, other than the point of inflection, of a cubic polynomial. This tangent will intersect the cubic at a second point, n. Draw a tangent line at this second point. The second tangent will intersect the cubic at a third point, p. Let A_1 be the area of the region between the first tangent line and the cubic and let A_2 be the area of the region between the cubic and the second tangent line. A general graph is given at the right.
   The interesting result is that the ratio A_2: A_1 is constant.



   (A) Find the ratio A_2: A_1.
   (B) Prove that the ratio is constant.

   Also see *Algebra in Motion* "Polynomial Surprises" #5
   by Audrey Weeks at www.calculusinmotion.com

4. Let R be the region enclosed by the parabola $y = x^2$ and the line normal to this parabola. For what value of $x > 0$ is the area of R a minimum? Write a function that gives the area of R, graph and analyze it.
5. Find the x-coordinate of the other two points where the line containing the points of inflection of any fourth degree polynomial intersects the polynomial.

Analytic Geometry Problems

6. Given the quadrilateral with vertices A(-5,2), B(11.3, 7.1), C(16.4,5) and D (0.1, –0.1))

   a. Show that *ABCD* is a parallelogram.
   b. Are the diagonals perpendicular? Show how you know.
   c. Show that the diagonals bisect each other.

7. Prove that any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. (Let (0,0) and (2*a*,0) be the endpoints as usual.)

8. Given the points $A(-3, 2)$ and $B(5, 4)$
    a. Find the length *AB*.
    b. Write an equation of the perpendicular bisector of $\overline{AB}$.
    c. Write an equation of the set of points (*x*, *y*) such that the sum of the distances from (*x*, *y*) to *A* and *B* is 9.

Trigonometry Problems

Solve the triangles:

9. **SSS**: Sides are 4.5, 6 and 8. Find angle opposite the side of 6

10. **SSA**: Angle 37.8, Side 8.75, next side 6 (or 2.5, or 10)

11. **ASA**: Angle 43.5, side 15; angle 50.7

"Old" Style Problems

12. The three numbers *a, h* and *b* form a *harmonic sequence* if their reciprocals, $\frac{1}{a}, \frac{1}{h}$ and $\frac{1}{b}$ form an arithmetic sequence. The number *h* is called the *harmonic mean* of *a* and *b*.
    (a) Find the harmonic mean of $\frac{1}{6}$ and $\frac{1}{12}$
    (b) Express *h* in terms of *a* and *b*.
    (c) Show that $\frac{1}{a}, \frac{1}{h}$ and $\frac{1}{b}$ form an arithmetic sequence.
13. Find two numbers such that the sum of each and its reciprocal is $\frac{17}{5}$. Show that your answers are correct.
14. Use the sequence operation to produce the sequence 3, 6, 9, 12, 15 as many different ways as you can.

15. Given the rational expression $\frac{x^3 - 9x^2 + 14x}{-x^2 - 3x + 10}$
    a. Reduce the expression.
    b. For what values of *x* (if any) is the original expression not defined? Explain how you know.
    c. For what values of *x* (if any) is the original expression not equal to the reduced expression? Explain how you know.

16. Factor over the rational numbers: $x^5 - 1$.

