

# AP Calculus Explorations for Teachers

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## RATE – ACCUMULATION PROBLEMS

The AP Calculus Course Description indicates that AB and BC calculus students should understand

- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
- Modeling rates of change.
- The interpretation of the derivative as a rate of change in varied applied contexts.
- The definite integral of a rate of change over an interval as the net amount of change in the quantity over the interval (accumulation function)

$$\int_a^b f'(t) dt = f(b) - f(a).$$

- Using the integral of a rate of change to give accumulated change
- The analytical and graphical analysis of functions defined by definite integrals  $f(x) = f(a) + \int_a^x f'(t) dt$  (accumulation functions).
- Using the Fundamental Theorem of Calculus to evaluate definite integrals.
- Finding specific antiderivatives using initial conditions.
- Model a written description of a physical situation with a function, differential equation or an integral.

These questions involve reading and interpreting words in a calculus context. The questions look “wordy” compared to some others. The key word is often “rate” or “rate of change” which students should immediately understand as a derivative. Thus, starting with a derivative they may be asked to do all the things one can do with a derivative, in the context of a real situation.

Another key idea is accumulation. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations. The integral of a rate of change gives the amount of change. The general form of the equation is

$F(x) = F(x_0) + \int_{x_0}^x f(t) dt$  where  $F$  is an antiderivative of  $f$ ,  $x = x_0$  is the initial time, and  $F(x_0)$  is the initial value.

### What students should know how to do:

- Be ready to read and apply; often these problems contain a lot of reading.
- Understand the question. It is often not necessary to do as much computation as it seems at first or as all the reading may seem to indicate.
- Use FTC may help differentiating  $F$ .
- Explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.

- There are problems with one rate and with 2 rates (in-out problems) of change work together
- Max/min and increasing / decreasing analysis.

**1. Exercise 1: 2000 AB 4** (No calculator allowed)

**The stem:**

- What are the key words in the stem?
- What's different about the rates other than one's constant and the other is not?
- What about the 30 gallons at  $t = 0$ ? What does that hint at?

**The parts:**

- There are two approaches to part (a). What are they?
- Part (b) is pretty easy, but it's really helping students think about the next part.
- Part (c) can be done by continuing either approach from part (a). Either way what do you end up with; what kind of function?
- Part (d) is a max/min problem based on the answer to (c).
  - How do you write a justification for the maximum? What needs to be said or shown?
  - How does the FTC fit into this?

Discuss the scoring standard for this question.

**2. Exercise 2: 2002 AB 2 / BC 2 "The Amusement Park" (Graphing Calculator allowed)**

**The stem:**

- Compare and contrast the given information in this question with 2000 AB 4 above.

**The parts:**

- Part (a): This (probably) could be done by either of the two methods used in the previous problem. Which do you prefer here? Why?
  
- Part (b): This was given only 1 point for the set up only. (The problem was that there are quite a few correct answers depending on when and where a student rounded off (correctly). Nevertheless, the set up itself is interesting.
  - Can you find more than one way to do the problem? What are they?
  
  - Which is "best"? Why?
  
- Part (c):
  - What does the expression for  $H(t)$  give in general terms and why does it give that?
  
  - The students were asked the meaning of  $H(17)$ . What is this testing?

- What theorem allows you to find  $H'(17)$  the “easy” way?
  
- There are 3 things that must be included in the interpretation of  $H'(17)$ .  
What are they?
  
- Part (d): There was no justification required here. The answer is found by the usual procedure of finding where  $H'(t) = 0$ . But let’s look further.
  - Graph the two given rate functions in the same window over the given domain.
  
  - Explain in terms of the 2 rate functions only how you can be sure their intersection,  $E(t) = L(t)$ , is the location of the maximum (and not a minimum) of  $H(t)$ .

Discuss the scoring standard for this question.

**3. Exercise 3:** Now you’re on your own. Do 2006 AB2 / BC2. Timed: 15 minutes.  
(Graphing Calculator allowed)

Discuss the scoring standard for this question.

**4. Exercise 4:** Many of the skills needed to answer these questions are taught in the 4-years of mathematics prior to studying the calculus. This exercise will give you some practice identifying these skills and show you how to start preparing your students for AP Calculus long before they start the class.

Return to 2006 AB 3 and write 2 questions based on this questions or its graph that would be suitable to use at each level listed below. You may, if necessary, revise the stem or graph slightly.

**Algebra 1**

**Geometry**

**Algebra 2**

**Trigonometry**

**Precalculus**

**8. Brainstorm:**

What can you have your STUDENTS doing on a regular basis so that they will master these concepts?

- 1.
- 2.
- 3.
- 4.
- 5.

### Index to AP Free-response questions Topic 5.

The questions themselves can be found at AP Central. Click on “The Exams” and then on “Exam Questions” then “Calculus AB” or “Calculus BC” in the table. or go to [http://apcentral.collegeboard.com/apc/members/exam/exam\\_questions/1997.html](http://apcentral.collegeboard.com/apc/members/exam/exam_questions/1997.html)

Topic 5: Accumulation & Rates		
Year & Question	GCA	Comments
1998 AB5-BC5	Yes	Draw graph, average value, accumulation
1999 AB3-BC3	Yes	From table: Riemann sum, Rolle’s theorem – MVT, average value
2000 AB4	No	2 solution methods: accumulation, max/min.
2002 AB2-BC2	Yes	“Amusement Park:” In-out, amounts, max/min, values
2002 AB2-BC2 B	Yes	Pollutants: max/min, amounts, accumulation
2003 AB3	Yes	From graph and table: difference quotient, max/min, Riemann sum, interpret integrals.
2003 AB2 B	Yes	Heating oil: In-out, inc/dec, accumulation, max/min
2004 AB1-BC1	Yes	Traffic flow: average value, average rate of change, accumulation, inc/dec.
2004 AB2 B	Yes	Mosquitoes: values, inc/dec, accumulation, max/min
2005 AB2	Yes	“Sandy Beach:” In-out, amounts, max/min, FTC
2005 AB2- BC2 B	Yes	Water tank: In-out, amount, max/min, inc/dec
2006 AB2-BC2	Yes	“Thomasville:” Average value and application

For similar explorations of other AP Calculus free-response type problems see

[http://www.linmcmullin.net/AP\\_Calculus\\_NEW.html](http://www.linmcmullin.net/AP_Calculus_NEW.html) or go to

[www.LinMcMullin.net](http://www.LinMcMullin.net) > RESOURCES > AP\* CALCULUS